

Math and Chess Workbook

Fractions, Decimals, Ratio, Equations and Problem Solving

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Note to teacher: _____ Date: _____

Note to parent: _____ Date: _____

Test Date: _____ Start: ___ / ___ / ___ / Finish: ___ / ___ / ___ / Assignment date: _____

Prime, relative prime, factor, factorization, multiple, GCF, LCM

A prime is a number that can only be a product of one and itself (Product Definition). A prime can also be explained as a number that is only divisible by one and itself (Division Definition). A number can either be a prime or nonprime (called composite) other than 0 and 1. The composite number can be a product of 2 or more sets of different factors, for example $4 = 1 \times 4$ or 2×2 .

The numbers 0 and 1 are neither prime nor composite. 0 is excluded by definition of prime and composite number. Although 1 could be considered as prime by the definition of prime but it is excluded simply because if 1 were considered as a prime number then it could be included in any sets of product of primes for example, $3 = 1 \times 3$, $4 = 1 \times 2^2$, this does not help solving problems other than it becomes redundancy for writing it every time.

When dealing “factor” question, use the Product Definition. When dealing with “divisor” question, use the Division Definition. The factor and divisor are synonyms.

Primes are important in finding the Product of Primes (POP or called factorization) for a number or to find the Greatest Common Factor (GCF) to reduce fractions to relative primes or Lowest Common Multiple (LCM) for some numbers and to find Lowest Common Denominator (LCD) for fractions additions and subtractions. The concept of prime can also be used in algebra to find the factors of polynomials.

How to find a prime

Since any number can be expressed as a product of primes, so we just have to divide that number by primes and see if it is divisible. If it is divisible then it is not a prime and if it is not divisible then it is a prime. For example, any number that is less than 100 can be expressed as a product of $2^a 3^b 5^c 7^d$, so if a 2-digit number (≤ 100) can not be divisible by either 2, 3, 5, or 7 then it is a prime, to do this division test, the largest possible prime needs to be tested for a number n is $\leq \sqrt{n}$.

For small number, one only needs to use the definition of prime to find out if a number could be a product of any other factors. For example, 4 could be a product of 1×4 but also of 2×2 . So 4 is not a prime.

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Prime or composite

Fill in the following with a yes or no.

Are the following numbers primes?	Think its factors.	Prime or not?
0		<input type="checkbox"/>
1		<input type="checkbox"/>
2	$2 = 1 \times 2$ (itself), any other factors?	<input type="checkbox"/> yes
3	$3 = 1 \times 3$ (itself), any other factors?	<input type="checkbox"/>
4	$4 = 1 \times 4$ (itself), also $4 = 2 \times 2$	<input type="checkbox"/>
5	$5 = 1 \times 5$, any other factors?	<input type="checkbox"/>
6		<input type="checkbox"/>
7		<input type="checkbox"/>
8		<input type="checkbox"/>
9		<input type="checkbox"/>
10		<input type="checkbox"/>
11		<input type="checkbox"/>
12		<input type="checkbox"/>

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Fill in the following with a yes or no.

Are the following numbers primes?	Think its factors.	Prime or not?
13		<input type="checkbox"/>
14		<input type="checkbox"/>
15		<input type="checkbox"/>
16		<input type="checkbox"/>
17		<input type="checkbox"/>
18		<input type="checkbox"/>
19		<input type="checkbox"/>
20		<input type="checkbox"/>
21		<input type="checkbox"/>
22		<input type="checkbox"/>
23		<input type="checkbox"/>
24		<input type="checkbox"/>
25		<input type="checkbox"/>

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Fill in the following with a yes or no.

Are the following numbers primes?	Think its factors.	Prime or not?
26		<input type="checkbox"/>
27		<input type="checkbox"/>
28		<input type="checkbox"/>
29		<input type="checkbox"/>
30		<input type="checkbox"/>
31		<input type="checkbox"/>
32		<input type="checkbox"/>
33		<input type="checkbox"/>
34		<input type="checkbox"/>
35		<input type="checkbox"/>
36		<input type="checkbox"/>
37		<input type="checkbox"/>
38		<input type="checkbox"/>

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Fill in the following with a yes or no.

Are the following numbers primes?	Think	Prime or not?
39		<input type="checkbox"/>
40		<input type="checkbox"/>
41		<input type="checkbox"/>
42		<input type="checkbox"/>
43		<input type="checkbox"/>
44		<input type="checkbox"/>
45		<input type="checkbox"/>
46		<input type="checkbox"/>
47		<input type="checkbox"/>
48		<input type="checkbox"/>
49		<input type="checkbox"/>
50		<input type="checkbox"/>
51		<input type="checkbox"/>

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Fill in the following with a yes or no.

Are the following numbers primes?	Think	Prime or not?
52		<input type="checkbox"/>
53		<input type="checkbox"/>
54		<input type="checkbox"/>
55		<input type="checkbox"/>
56		<input type="checkbox"/>
57		<input type="checkbox"/>
58		<input type="checkbox"/>
59		<input type="checkbox"/>
60		<input type="checkbox"/>
61		<input type="checkbox"/>
62		<input type="checkbox"/>
63		<input type="checkbox"/>
64		<input type="checkbox"/>

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Fill in the following with a yes or no.

Are the following numbers primes?	Think	Prime or not?
65		<input type="checkbox"/>
66		<input type="checkbox"/>
67		<input type="checkbox"/>
68		<input type="checkbox"/>
69		<input type="checkbox"/>
70		<input type="checkbox"/>
71		<input type="checkbox"/>
72		<input type="checkbox"/>
73		<input type="checkbox"/>
74		<input type="checkbox"/>
75		<input type="checkbox"/>
76		<input type="checkbox"/>
77		<input type="checkbox"/>

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Fill in the following with a yes or no.

Are the following numbers primes?	Think	Prime or not?
78		<input type="checkbox"/>
79		<input type="checkbox"/>
80		<input type="checkbox"/>
81		<input type="checkbox"/>
82		<input type="checkbox"/>
83		<input type="checkbox"/>
84		<input type="checkbox"/>
85		<input type="checkbox"/>
86		<input type="checkbox"/>
87		<input type="checkbox"/>
88		<input type="checkbox"/>
89		<input type="checkbox"/>
90		<input type="checkbox"/>

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Fill in the following with a yes or no.

Are the following numbers primes?	Think	Prime or not?
91		<input type="checkbox"/>
92		<input type="checkbox"/>
93		<input type="checkbox"/>
94		<input type="checkbox"/>
95		<input type="checkbox"/>
96		<input type="checkbox"/>
97		<input type="checkbox"/>
98		<input type="checkbox"/>
99		<input type="checkbox"/>
100		<input type="checkbox"/>
101		<input type="checkbox"/>
102		<input type="checkbox"/>
103		<input type="checkbox"/>

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The primes less than 100 can be memorized with a pattern that is all primes end with one of these numbers: 1, 3, 7, 9 other than the first 4 primes: 2, 3, 5, 7.

2, 3, 5, 7

11, 13, 17, 19

23, 29

31, 37

41, 43, 47

53, 59

61, 67

71, 73, 79

83, 89

97

Relative prime

Relative primes are numbers having only 1 as their greatest common factor (GCF) such as 17 and 4. 17 and 4 are relative primes to each other. The concept of relative prime is very important in finding GCF, LCM and the rational expressions and trinomial factoring in the secondary school.

	What are the common factors?	Relative primes? yes or no.
6, 7	_____	_____
12, 14	_____	_____
31, 35	_____	_____
17, 51	_____	_____
24, 48	_____	_____
6, 18	_____	_____
3, 9	_____	_____

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Interesting primes

There are 25 prime numbers in the numbers from 1 to 100.

All the primes we know so far are odd numbers except 2.

An even number (other than 2) can be expressed as a sum of two primes (Goldbach's conjecture).

Emirps (**palindromic prime**) means there is a pair of primes that when each of its two digits is switched, it becomes the other prime. There are 5 pairs of Emirps in numbers < 100.

11, 13, 17, 37, 79
11, 31, 71, 73, 97

(Combinations of 1, 3, 7, 9)

Twin Primes (8 pairs less than 100) are a pair of two consecutive primes and their difference is 2. There are 8 pairs of twin primes that are less than 100.

3, 5, 11, 17, 29, 41, 59, 71
5, 7, 13, 19, 31, 43, 61, 73

A Mersenne prime is a prime that can be written in the form of $2^p - 1$, where p is a prime. The largest Mersenne prime discovered so far is $2^{13466917} - 1$ as of December 2001 and it contains 4,053,946 digits.

Prime Problems

Prime problems	Answers
What is the smallest prime number?	_____
What is the smallest composite number?	_____
What is the smallest even prime number?	_____
What is the largest even prime number?	_____
What is the smallest odd prime number?	_____

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Prime Problems

Prime problems	Answers
What is the greatest prime number that is less than 100?	_____
What is the greatest prime number that is less than 50?	_____
What is the greatest prime number that is less than 40?	_____
What is the smallest prime number that is greater than 80?	_____
What is the smallest prime number that is greater than 90?	_____
What is the smallest prime number that is greater than 150?	_____
What is the sum of the first ten prime numbers?	_____
What is the product of prime numbers between 70 and 80?	_____
What is the smallest prime number that is greater than 80?	_____
Write all possible answers of 30 as the sum of two distinct primes.	_____
What is the largest prime number whose square is less than 1000?	_____

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Prime Problems

Prime problems	Answers
What is the greatest prime number whose square is less than 10000?	_____
What is the greatest prime number that is less than 1000?	_____
In how many ways can the number 24 be expressed as the sum of 2 primes?	_____
In how many ways can the number 48 be expressed as the sum of 2 primes?	_____
Twin prime pairs (see previous Interesting primes section) are in the form of n and $n+2$. List all twin primes below 100.	_____
How many pairs of two-digit <i>emirps</i> exist such that each number in the pair is greater than 11?	_____
How many prime numbers less than 100 have a units digit of 1?	_____
The digits 1, 3, 7 are each used exactly once to form three-digit primes. List all the possible primes.	_____
The digits 1, 2, 5 are each used exactly once to form three-digit primes. List all the possible primes.	_____
Write all possible answers of 40 as the sum of two distinct primes.	_____
What is the largest prime number whose square is less than 100?	_____

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Factor, factorization, factor tree, repeated division

Every composite number can always be a product of at least 2 prime numbers; these 2 different numbers are called factors. So a factor is a whole number that defines another number evenly. A prime number can be a product of 1 and itself, these 1 and “itself” number are also called factors. Factors are found by reversing the thinking process of multiplication. Factors come in pairs of different numbers other than doubles such $36 = 6 \times 6$. Order of factors does not matter. For example, $2 = 1 \times 2$, the right hand side 1 and 2 are called factors (What 2 numbers multiplied together is 2?). A number of 12 can have 1, 2, 3, 4, 6, and 12 as its factors. These factors may not be all primes.

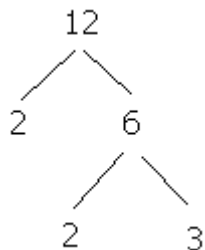
$$12 = 1 \times 12$$

$$12 = 2 \times 6$$

$$12 = 3 \times 4$$

Quite often, we are interested in finding only the prime factors of a number (factorization). By knowing the primes, we can find the lowest common multiple (LCM) and the greatest common factor (GCF). By knowing GCF and LCM, we can work with fractions. So the knowledge of prime is very important and we even use this prime concept in the secondary school to do factorization for trinomials.

Factor tree method and repeated division method are two very practical methods in finding prime factors for 2 or more numbers. For example, the prime factors of 12 can be found by using the following factor tree method.



The results of above end branches numbers 2, 2, 3 are all prime factors of 12. 12 can be expressed as a product of prime numbers, so the Product of Prime (POP) for $12 = 2 \times 2 \times 3 = 2^2 \times 3$.

The factor tree ends when the end numbers are all primes. The result of 2 factors multiplied together always is the product of the previous number. Do not use 1 as the factor.

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The following division method can also be used to find the prime factors for 12 starting with the smallest prime divisor until the prime quotient is reached.

$$\begin{array}{r} 2 \overline{)12} \\ 2 \overline{)6} \\ \underline{3} \end{array}$$

POP for 12 = $2 \times 2 \times 3$

The repeated division method is superior to the factor tree method in finding out the GCM and LCM. The factor tree method may offer better visualization and as a concept teaching tool when finding common factors, GCF, and LCM. To find common factors by using factor trees, the results of factor trees need to be analyzed to get results. However, the repeated division not only let students find LCM and GCF, it also intuitively trains students to “see” what is the common factor by visualization or trial division. This skill learned is very important in secondary school when students are asked to factorize common factor, for example $2x^2 + 3x$ since the repeated division method requires students to find the common factor for 2 or 3 numbers at once.

The Product of Primes (POP) is useful in finding out the number of factors, see author’s another publication: Math Contest Preparation, ISBN0-9683967-2-0 for details.

Factor tree and repeated division

Use factor tree and the repeated division methods to find prime factors for the following numbers.

Factor tree	Repeated division	Factor tree	Repeated division
16 $\swarrow \searrow$	$\quad) 16$	24 $\swarrow \searrow$	$\quad) 24$
$16 = \underline{\hspace{2cm}}$	$16 = \underline{\hspace{2cm}}$	$24 = \underline{\hspace{2cm}}$	$24 = \underline{\hspace{2cm}}$

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Factor tree and repeated division

Use factor tree and the repeated division methods to find prime factors for the following numbers.

Factor tree	Repeated division	Factor tree	Repeated division
18 $\swarrow \searrow$	$\underline{) 18}$	32 $\swarrow \searrow$	$\underline{) 32}$
$18 = \underline{\hspace{2cm}}$	$18 = \underline{\hspace{2cm}}$	$32 = \underline{\hspace{2cm}}$	$32 = \underline{\hspace{2cm}}$

Factor tree	Repeated division	Factor tree	Repeated division
46 $\swarrow \searrow$	$\underline{) 46}$	54 $\swarrow \searrow$	$\underline{) 54}$
$46 = \underline{\hspace{2cm}}$	$46 = \underline{\hspace{2cm}}$	$54 = \underline{\hspace{2cm}}$	$54 = \underline{\hspace{2cm}}$

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Multiples list and LCM

The multiples of a number can be found by multiplying the given number by 1, by 2, by 3 and so on. For example, the first 7 multiples of 3 are: 3, 6, 9, 12, 15, 18, and 21. The Lowest Common Multiple (LCM) is the smallest number that is a multiple of both (or more) numbers. Multiples list could be used to find LCM, see the following example in finding LCM for 5 and 20.

Numbers	Multiples list	Common Multiples	Lowest Common Multiple (LCM)	Greatest Common Multiple
5	5, 10, 15, 20 , 25, 30, 35, 40 , 45, 50, 55, 60 , 65, 70, 75, 80 , 85, 90, 95, 100 , 105, 110, 115, 120, 125, 130, 135, 140 , 145, 150, 155, 160 , ...	20, 40, 60, 80, 100, 120, 140, 160 (Note that all common multiples are multiples of 20.)	20	Does not exist
20	20, 40, 60, 80, 100, 120, 140, 160, ...			

Numbers	Next 5 multiples	Common Multiples	LCM
3	<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>	<input type="text"/>	<input type="text"/>
4	<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>		

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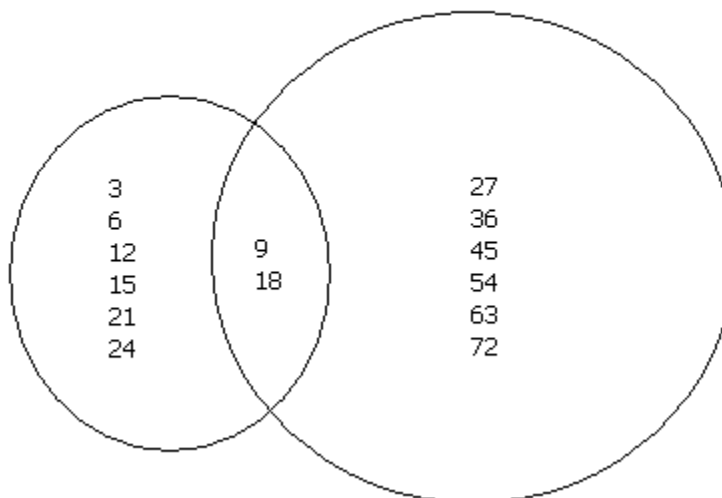
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The following shows how to use diagram to find the common multiples between 3 (up to 24) and 9 (up to 72). Can you list common multiples shown below between 3 and 9? Answer _____



Fill in the following with a number.

Numbers	Multiples	Common multiples	LCM
12	<input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/>	<input type="text"/> , <input type="text"/>	<input type="text"/>
16	<input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/>		

Fill in the following with a number.

Numbers	Multiples	Common multiples	LCM
8	<input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/>	<input type="text"/>	<input type="text"/>
18	<input type="text"/> , <input type="text"/> , <input type="text"/> , <input type="text"/>		

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Doubles and triples

Fill in the following with a number.

Numbers	Doubles (twice of)	Triples
4	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>
21	<input type="checkbox"/>	<input type="checkbox"/>
53	<input type="checkbox"/>	<input type="checkbox"/>

Double of 2 plus triple of 3 =

Triple of 3 minus double of 3 =

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Multiple Problems

The first number is 12 and the second number is 4 less than the first, the third number is 4 more than twice of the first. What are the second number and the third number?

The first number is 4 and the second number is four times the first, what is their sum?

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Using factor list to find common factors and GCF

A common factor of two numbers is a factor that can divide evenly into both numbers. A common factor for two numbers is a factor existing in both numbers. We can also say that a common factor of 2 numbers is a factor that can go into both numbers. The Greatest Common Factor (GCF) is the largest factor that is a factor of both numbers.

	Factor list	Common factors between 12 and 16	Greatest Common Factor (GCF)
Numbers	Any Product = factor1 \times factor2 (Factors are in pairs, but may not be primes)		
12	12 can be a product of the following different factors: $12 = 1 \times 12$ $12 = 2 \times 6$ $12 = 3 \times 4$ 12 has factors <u>1</u> , <u>2</u> , 3, <u>4</u> , 6, 12	1, 2, 4	<input type="text"/>
16	16 can be a product of the following different factors: $16 = 1 \times 16$ $16 = 2 \times 8$ $16 = 4 \times 4$ 16 has factors <u>1</u> , <u>2</u> , <u>4</u> , 8, 16	1, 2, 4	<input type="text"/>

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Using factor list to find common factors and GCF

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Numbers	Factor list	Common factors between 24 and 32	Greatest Common Factor (GCF)
16	$16 = \square \times \square$ $16 = \square \times \square$ $16 = \square \times \square$	$\square \square \square$	\square
24	$24 = \square \times \square$ $24 = \square \times \square$ $24 = \square \times \square$ $24 = \square \times \square$	$\square \square \square$	\square

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A common factor of two numbers is a factor that can divide both numbers evenly. A common factor for two numbers is a factor existing in both numbers. We can also say that a common factor of 2 numbers is a factor that can go into both numbers. The Greatest Common Factor (GCF) is the largest factor that is a factor of both numbers.

Numbers	Factor list	Common factors between 24 and 32	Greatest Common Factor (GCF)
24	$24 = \square \times \square$ $24 = \square \times \square$ $24 = \square \times \square$ $24 = \square \times \square$	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> 	<input type="checkbox"/>
32	$32 = \square \times \square$ $32 = \square \times \square$ $32 = \square \times \square$	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> 	<input type="checkbox"/>

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Using factor tree to find GCF and LCM of 2 numbers

A factor tree is a method of finding all primes numbers of a number such that the product of all primes equal to that number. In other words, factor tree can be used to find the number of prime factors of a number. The Product of Primes (POP) = Number. For example, $12 = 2 \times 2 \times 3$. 12 is the product of all primes (POP) and it is also called prime factorization, factorize a number to a product of all primes. The Lowest Common Multiple (LCM) is the smallest number that is a multiple of both (or more) numbers. The Greatest Comm Factor (GCF) is the greatest common factor that exists in both (or more) numbers.

Step 1: Find 2 factors, which multiplied together, are 12. (Do not use 1 since $1 \times \text{number} = \text{number}$.)

Step 2: Find another pair of numbers, which multiplied equals to the previous non-prime number.

Step 3: Repeat step 2 until all numbers become primes.

The result of all primes can be used to find the Greatest Common Factor (GCF) and the Lowest Common Multiple (LCM).

Numbers	Factor Tree	GCF	LCM
12	<pre> graph TD 12 --- 2 12 --- 6 6 --- 2 6 --- 3 </pre>	$12 = 2 \times 2 \times 3$ $16 = 2 \times 2 \times 2 \times 2$ 2×2 appears twice in both 12 and 16, so 2×2 is GCF for both 12 and 16. The GCF is $2 \times 2 = 4$. $12 = 2^2 \times 3$	2 and 3 are prime factors in either numbers 12 or 16. LCM is product of primes with the largest number of occurrence (highest exponents) so $2 \times 2 \times 2 \times 2 \times 3 = 48$.
16	<pre> graph TD 16 --- 2 16 --- 8 8 --- 2 8 --- 4 4 --- 2 4 --- 2 </pre>	$12 = 2^2 \times 3$ $16 = 2^4$ Examine the number in exponential form, it clearly shows that 2^2 is GCM, but the elementary students may not understand.	By examine $12 = 2^2 \times 3$ and $16 = 2^4$, we know that numbers appear with the highest exponents in either number are 2^4 and 3, the product is the GCF = $2^4 \times 3 = 48$

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Numbers	Factor Tree	GCF	LCM
24			
32			

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Step 1: Find 2 factors, which multiplied together, are 12. (Do not use 1 since $1 \times \text{number} = \text{number}$.)

Step 2: Find another pair of numbers, which multiplied equals to the previous non-prime number.

Step 3: Repeat step 2 until all numbers become primes.

The result of all primes can be used to find the Greatest Common Factor (GCF) and the Lowest Common Multiple (LCM).

Numbers	Factor Tree	GCF	LCM
36			
48			

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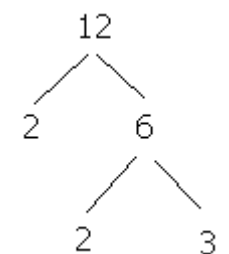
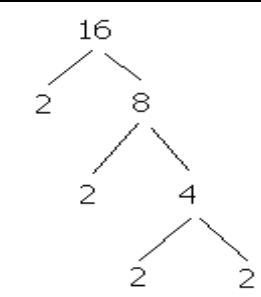
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Methods of finding Greatest Common Factor (GCF) of 2 numbers

To find GCF or LCM for large numbers (3 or more digits), use Euclidean algorithm (Details see Math Contest Preparation, ISBN 0-9683967-2-0).

	Factor list	Factor tree	Repeated division	GCF = largest factor of the smaller number if it can divide the larger number
Numbers	Any Product = factor1 × factor2 (Factors are in pairs, but may not be primes)	Start with the smallest prime.	Start with the largest divisor.	For example 2 and 6, the GCF is 2 since it is divisible by 6 and the largest factor of 2 is 2.
12	12 can be a product of the following different factors: $12 = 1 \times 12$ $12 = 2 \times 6$ $12 = 3 \times 4$ 12 has factors 1, 2, 3, 4, 6, 12	 <p>POP for 12 $= 2 \times 2 \times 3$ $= 2^2 \times 3$</p>	$4 \overline{)12, 16}$ $\quad 3, 4$ GCF for 12 $= 4$ If you cannot find 4 then start with 2 is okay too. $2 \overline{)12, 16}$ $2 \overline{)6, 8}$ $\quad 3, 4$	For example 3 and 9, the GCF is 3 since it is divisible by 9 and the largest factor of 3 is 3.
16	16 can be a product of the following different factors: $16 = 1 \times 16$ $16 = 2 \times 8$ $16 = 4 \times 4$ 16 has factors 1, 2, 4, 8, 16	 <p>POP for 16 $= 2 \times 2 \times 2 \times 2$ $= 2^4$ GCF = $2 \times 2 = 4$ (Note: 2^2 are factors in both numbers.)</p>	Continue the division until all numbers are relative primes.	For example 12 and 48, the GCF is 12 since it is divisible by 48.

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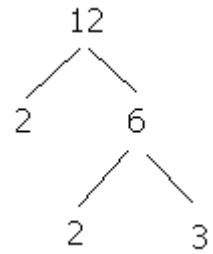
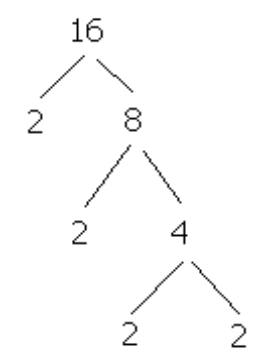
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Methods of finding Lowest Common Multiple (LCM) of 2 numbers

	Multiple of larger number or product of relative primes	Factor Tree	Repeated division	
Numbers	When 2 numbers are small.	Start with the smallest prime.	Start with the largest divisor.	
12	<p>Are these 2 numbers relative primes? No? Follow the following procedure:</p> <ol style="list-style-type: none"> 1. Is 16 divisible by 12, no? Continue with the next multiple of 16 that is 32. 2. Is 32 divisible by 12, No? Continue with the next multiple of 16 that is 48. 3. Is 48 divisible by 12?, Yes. The LCM is 48. 	 <p>POP for 12 $= 2 \times 2 \times 3$ $= 2^2 \times 3$</p>	$\begin{array}{r} 4 \overline{)12, 16} \\ \underline{3, 4} \end{array}$ <p>GCF for 12 $= 4$ If you can not find 4 then start with 2 is okay too.</p> $\begin{array}{r} 2 \overline{)12, 16} \\ \underline{2, 8} \\ \underline{3, 4} \end{array}$	$\text{LCM} = 4 \times 3 \times 4 = 48$ $= \text{GCF} \times 3 \times 4$
16	<p>If both numbers are relative primes such as 2, 5, and then 2×5 is the LCM. The LCM for 5 and 8 is $5 \times 8 = 40$. The LCM for 4 and 9 is $4 \times 9 = 36$.</p>	 <p>POP for 16 $= 2 \times 2 \times 2 \times 2$ $= 2^4$ $\text{LCM} = 2^2 \times 2 \times 2$ $\times 3 = 48 = \text{GCF} (4)$ $\times \text{no-match factors}$</p>	<p>Continue the division until all numbers are relative primes.</p>	

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Find GCF and LCM of the following numbers.

Numbers	GCF (repeated division method)	LCM (repeated division method)	LCM (larger number or multiple of the larger number)
9, 15	$\begin{array}{r}) 9, 15 \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) ______ \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
12, 32	$\begin{array}{r}) 12, 32 \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) ______ \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
6, 15	$\begin{array}{r}) 6, 15 \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) ______ \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
45, 36	$\begin{array}{r}) 45, 36 \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) ______ \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
12, 42	$\begin{array}{r}) 12, 42 \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) ______ \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____

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Find GCF and LCM of the following numbers.

Numbers	GCF (repeated division method)	LCM (repeated division method)	LCM (larger number or multiple of the larger number)
36, 9	$\begin{array}{r}) \underline{36, 9} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
7, 42	$\begin{array}{r}) \underline{7, 42} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
36, 45	$\begin{array}{r}) \underline{36, 45} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
9, 6	$\begin{array}{r}) \underline{9, 6} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
30, 25	$\begin{array}{r}) \underline{30, 25} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____

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Find GCF and LCM of the following numbers.

Numbers	GCF (repeated division method)	LCM (repeated division method)	LCM (larger number or multiple of the larger number)
9, 12	$\begin{array}{r}) \underline{9, 12} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
4, 32	$\begin{array}{r}) \underline{4, 32} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
6, 16	$\begin{array}{r}) \underline{6, 16} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
5, 6	$\begin{array}{r}) \underline{5, 6} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____
$2^2 \times 3, 2^2 \times 7$	$\begin{array}{r}) \underline{2^2 \times 3, 2^2 \times 7} \\ \hline \end{array}$ GCF = _____	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \hline \end{array}$ LCM = _____	Multiple of larger number = $\square \times \square$ LCM = _____

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Method of finding GCF and LCM for 3 or more large numbers

Both “multiple of larger number or the product of relative primes” method and the factor tree method become clumsy in finding LCM of 3 numbers especially when the numbers are large. The “largest factor of the small number” method in finding GCF of 3 numbers becomes difficult. The recommended method for finding GCF or LCM for 3 or more numbers is the repeated division method.

Repeated division Method when 3 numbers are large.	Comments
$\begin{array}{r} 2) \underline{108, 84, 78} \\ 3) \underline{54, 42, 39} \\ 2) \underline{18, 14, 13} \\ 9, 7, 13 \end{array}$ <p>GCF = $2 \times 3 \times 2 = 12$ LCM = $2 \times 3 \times 2 \times 9 \times 7 \times 13$ = 9828</p>	<p>Continue the division until all quotients are relative primes.</p> <p>Since 18 and 14 are not relative primes, so the division continues. Just bring down 13 to continue the division. 9, 7, 13 are all relative primes so the division stops.</p>
$\begin{array}{r} 2) \underline{24, 47, 34} \\ 12, 47, 17 \end{array}$ <p>GCF = 1 LCM = $2 \times 12 \times 47 \times 17$ = 19176</p>	<p>47 is not divisible by 2, so just bring down 47. 12, 47, and 17 are all relative primes so the division stops.</p>
$\begin{array}{r} 13) \underline{91, 52} \\ 7, 4 \end{array}$ <p>GCF = 13 LCM = $13 \times 7 \times 4 = 364$</p>	<p>71 is not divisible by 2, so 2 is not GCF. GCF = 1. Bring down 71. LCM = $2 \times 6 \times 71$.</p>
$\begin{array}{r} 2) \underline{2, 12, 71} \\ 1, 6, 71 \end{array}$ <p>GCF = 1 LCM = $2 \times 6 \times 71 = 2272$</p>	<p>71 is not divisible by 2, so 2 is not GCF. GCF = 1. Bring down 71. LCM = $2 \times 6 \times 71$.</p>

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Method of finding GCF and LCM for 3 or more small numbers

multiples of larger number or the product of relative primes when 3 numbers are small.	Comments
2, 5, 6 The GCF is 1. The LCM is $6 \times 5 = 30$.	Since 6 is the multiple of 2 so the question is really like to find the LCM between 5 and 6. LCM for 5 and 6 is $5 \times 6 = 30$ because 5 and 6 are relative primes.
4, 12, 15 The GCF is 1. The LCM is $4 \times 15 = 60$.	Since 12 is the multiple of 4 so the question is really to find the LCM between 12 and 15. Use the multiple of $4 \times 15 = 60$ which is divisible by 12, so the LCM is 60. The GCF = 1

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Find GCF and LCM of the following 3 numbers.

numbers	GCF (repeated division)	LCM (repeated division)
9, 12, 2	$\begin{array}{r} 1 \) \ 9, 12, 2 \\ \underline{ 9, 12, 2} \\ \end{array}$ <p style="text-align: center;">GCF = 1</p>	$\begin{array}{r} 3 \) \ 9, 12, 2 \\ \underline{ 9, 12, 2} \\ \\ 2 \) \ 3, 4, 2 \\ \underline{ 3, 4, 2} \\ \\ 3, 2, 1 \end{array}$ <p style="text-align: center;">LCM = $3 \times 2 \times 3 \times 2$ = 36</p>
4, 32, 6	$\begin{array}{r} \) \ 4, 32, 6 \end{array}$ <p style="text-align: center;">GCF =</p>	<p style="text-align: center;">LCM =</p>
6, 16, 5	$\begin{array}{r} \) \ ______ \end{array}$ <p style="text-align: center;">GCF =</p>	<p style="text-align: center;">LCM =</p>
36, 90, 20	$\begin{array}{r} \) \ ______ \end{array}$ <p style="text-align: center;">GCF =</p>	<p style="text-align: center;">LCM =</p>

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Find GCF and LCM of the following 3 numbers.

numbers	GCF (repeated division)	LCM (repeated division)
45, 60, 10	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \end{array}$ GCF =	 LCM =
12, 36, 2	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \end{array}$ GCF =	 LCM =
36, 8, 4	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \end{array}$ GCF =	 LCM =
7, 64, 8	$\begin{array}{r}) \underline{\hspace{2cm}} \\ \end{array}$ GCF =	 LCM =

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LCM problems

1. Meghan is having a party and she knows there could be 4, 5, or 6 guests showing up.

She wants to make sure each guest would have the same number of candies, what would be the least number of candies she has to buy? _____

If 4 guests showed up then how many candies each guest could have equally? _____

If 5 guests showed up then how many candies each guest could have equally? _____

If 6 guests showed up then how many candies each guest could have equally? _____

2. Nicol runs the track field every 7 minutes, Angela runs the same field every 9 minutes and Author runs every 6 minutes. If they start running at the same time what time will they meet again next time?

They will meet again in _____ minutes.

They will meet again after Nicol runs _____ laps.

They will meet again after Angela runs _____ laps.

They will meet again after Author runs _____ laps.

3. Joan goes to math and chess club ever 4 days, John goes to the same club every 5 days and Jessica goes every 8 days. If they meet today when will they meet altogether next time?

They will meet again in _____ days.

They will meet again after Joan has gone to the club ____ times.

They will meet again after John has gone to the club ____ times.

They will meet again after Jessica has gone to the club ____ times.

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GCF problems

1. Amy bought 12 pencils, 15 erasers, and 24 ballpoint pens. She wants to place them in loot bags with equal number of items in each bag. What is the largest number of loot bags she has to prepare?

Amy could have _____ loot bags.

Each bag will have _____ pencils, _____ erasers and _____ ballpoint pens.

2. Ashley bought 28 hockey cards, 16 basketball cards and 24 baseball cards. She likes to give these cards away to some of her friends in such way that each friend would receive equal number of hockey cards, equal number of basketball cards, and equal number of baseball cards. How many of friends will receive her gifts?

_____ of Ashley's friends will receive her gifts.

Each friend will receive _____ hockey cards, _____ basketball cards, and _____ baseball cards.

3. Both Stanley and Edward went to this year's math and chess class reunion. Stanley could only go to the next reunion if it were to be held every 3 years and Edward could go if it were to be held every 5 years, what year should the next reunion to be held so both Stanley and Edward would meet again?

If the next reunion is to be held in _____ years later from this year, Stanley and Edward will meet again.

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GCF and LCM problems

1. What is the greatest common factor of all 2-digit palindromes?
2. What is the largest three-digit number divisible by 3, 4, and 5?
3. What is the greatest divisor of 35 and 65?
4. Two measuring tapes have the same length but with different scales. Tape A has 1771 marks and tape B has 1785 marks and each tape has a different kind of measuring scale. Both tapes start with mark 0. What is the mark on tape A and the mark on tape B when both tapes line up exactly the first time after sale mark 0?