

Math Contest Preparation

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Shortcut: Primes

Explanation

A prime is a number that can only be a product of one and itself (Product Definition). Therefore, if a prime is represented by a rectangular diagram, it only has one row. A prime can also be explained as a number that is only divisible by one and itself (Division Definition). A number can only be a prime or non prime (called composite) except 0 and 1. The composite number can be a product of 2 or more sets of different factors, for example $4 = 1 \times 4$ or 2×2 .

The numbers 0 and 1 are neither prime nor composite. 0 is excluded by definition of prime and composite number. Although 1 could be considered as prime by the definition of prime but it is excluded simply because if 1 were considered as a prime number then it could be included in any POP for example, $3 = 1 \times 3$, $4 = 1 \times 2^2$, this does not help solving problems other than it becomes redundancy for writing it every time.

1 is also excluded in the computation of exponential function such as $y = a^x$, $a \neq 1$ simply because 1 to any power is still 1 so there is no need to solve x .

When dealing “factor” question, use the Product Definition. When dealing with “divisor” question, use the Division Definition.

Quite often, we will see primes that are less than 100 in contest questions, so you are encouraged to memorize those primes that are less than 100. The primes less than 100 can be memorized with a pattern described below. Divide 100 primes into 3 groups as follows:

Group 1

2, 3, 5, 7
11, 13, 17, 19

Group 2

23, 29
31, 37
41, 43, 47
53, 59
61, 67
71, 73, 79

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Group 3

83, 89

97

~~1~~-The pattern of the middle group is that the unit-digit follows the pattern of 3, 9; 1, 7; 1, 3, 7; 3, 9; 1, 7; 1, 3, 9. ~~⊖~~Other than the first four primes 2, 3, 5, 7, all other primes end with one of these numbers: 1, 3, 7, and 9.

~~2~~-There are 25 prime numbers in the numbers from 1 to 100.

~~3~~-All the primes we know so far are odd numbers except 2.

An even number (other than 2) can be expressed as a sum of two primes (Goldbach's conjecture).

Since any number can be expressed as a product of primes, so we just have to divide that number by primes and see if it is divisible. If it is divisible then it is not a prime and if it is not divisible then it is a prime. For example, any number that is less than 100 can be expressed as a product of $2^a 3^b 5^c 7^d$, so if a 2-digit number (≤ 100) can not be divisible by either 2, 3, 5, or 7 then it is a prime. The largest possible prime needs to be tested for a number is \sqrt{n} .

~~5~~-Emirps (palindromic prime) means there is a pair of primes that when each of its two digits is switched, it becomes the other prime. There are 5 pairs of Emirps in numbers < 100 .

~~_____~~ 11₂ _____ 13₂ _____ 17₂ _____ 37₂ _____ 79₂
~~_____~~ 11₂ _____ 31₂ _____ 71₂ _____ 73₂ _____ 97₂

~~_____~~-(Combinations of 1, 3, 7, 9)

~~6~~-Twin Primes (8 pairs less than 100) are a pair of two consecutive primes. There are 8 pairs of twin primes in the numbers < 100 .

~~_____~~ 3₂ _____ 5₂ _____ 11₂ _____ 17₂ _____ 29₂ _____ 41₂ _____ 59₂ _____ 71₂

~~_____~~ 5₂ _____ 7₂ _____ 13₂ _____ 19₂ _____ 31₂ _____ 43₂ _____ 61₂ _____ 73₂

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A Mersenne prime is a prime that can be written in the form of $2^p - 1$, where p is a prime. The largest Mersenne prime discovered so far is $2^{13466917} - 1$ as of December 2001 and it contains 4,053,946 digits.

8. —A pair of relative primes are two numbers a and b when $\text{GCF}(a,b) = 1$ then we say a and b are relative primes (a and b are not necessary prime numbers.). For example, $(9,16)$, $(4,9)$, $(2,5)$ are all relative primes. In fraction, the simplified form of numerator and denominator are relative primes.

A prime number can always be expressed as $6n + 1$ or $6n - 1$, for example, $89 = 6 \times 15 - 1$, $73 = 6 \times 12 + 1$, but it is only a necessary condition to be a prime number since the reverse may not be true. For example, $6(6) - 1 = 35$ which is not a prime.

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Exercises

~~1~~.1. Find the smallest Mersenne prime that is greater than 100.

~~2~~.2. To test if 307 is a prime, we can test see if 307 is divisible by 2, 3, 5, 7, 11, 13, and ____ .
(hint $\sqrt{307} = 17.xx$)

~~3~~.3. How many pairs of Emirps are < 100 but > 11 ?

4. A palindromic prime is a prime number that is also prime when its digits are reversed. What is the smallest palindromic prime greater than 31?

5. What is the positive difference of the smallest odd prime and the largest even prime?

6. What is the largest 2-digit prime number whose digits are also each prime?

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7. How many primes less than 100 have 1 as their units digit?

8. How many primes less than 100 have 3 as their units digit?

9. How many primes less than 100 have 7 as their units digit?

10. How many primes less than 100 have 9 as their units digit?

11. What is 50% of the sum of the first 9 prime numbers?

12. How many trailing zeros of the product of all the prime numbers less than 100?

13. What is the sum of the least one-digit prime and the greatest 2-digit prime?

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14. What is the product of the smallest one-digit prime and the smallest two-digit prime?

15. What is the largest 2-digit prime number that the product of its digits is also a prime number?

16. What is the least prime number, which is 21 greater than another prime number?

17. One of the Goldbach's conjectures states that every even number greater than five can be expressed as the sum of two primes. What is the greatest possible product of two distinct primes whose sum is 10?

18. In the equation $x(y + z) = 209$, where x, y, z are prime numbers. What are these prime numbers?